Mathematical structuralism: internal and external

Although the general motto that mathematics is a study of structures is rarely contested at present, there still remains a noticeable divide between two broad groups of philosophers of mathematics. First, there are the more philosophically inclined authors (Hellman, Resnik, Shapiro and others), who explain that mathematics is not about isolated objects with distinct internal features but rather about whole structures, patterns or structured systems with "objects" being only specified by its outer relations to the rest of the whole structure. The second group are more mathematically oriented authors (Awodey, Landry, McLarty and others) for whom mathematical structuralism simply means mathematics as presented in the language of category theory. The relations of the two groups is characterized by mutual suspicion, misunderstanding, even ignorance. A recent evidence of the gap is Awodey's (2014) admission that the notion of structure as presumed by the two groups "differs radically".

My claim is that the divide, though understandable and based on actual differences, is still not unbridgeable. To be more specific, the difference stems from the way the two groups address structures themselves. From the point of view of category theorists, the philosophical structuralists treat the structures as, well, internally structured. For the category theorists, though, this represents a breach of the central tent of mathematical structuralism: to always address mathematical entities via its external relations only and not according to its supposed internal structure. Their opposition is often presented along the lines of claiming that internal structuralism always introduces some (externally) structurally irrelevant features to the picture. True enough, this was exactly the reason why mathematical structuralism was formulated in the first place: not to get distracted by any internal features of the mathematical entities and to study only their external mutual relations. The only "clean way" to speak about structures themselves thus seems to address them also strictly externally, in the way category theory does.

Yet, the division into external and internal structuralism is not so clear-cut as it might appear at first sight. In fact, the fixed frame of any category theory (formal) narative is always a definite category which itself is an internally structured entity. Although any studied mathematical structure (say natural numbers) is addressed (as an object) strictly by its external relations (morphisms), it is only within a pre-specified category that any such framing may take place. Structure identification, structure isomorphism and all the likes are always (a specific) category related concepts. So, category theory structuralism is in some sense internal too. On the other hand, if a philosophical structuralist wants to speak about the whole structure as such, they have to "step outside of it" and address it from the external-relations-point-of-view. They have to become, albeit only implicitly, external relationist too. In fact, what I want to demonstrate is that there exists a sort of general dualism between internal and external structuralism: any (externally) structural system determines (implicit) internal structural features of its objects and, on the other hand, internally structured object constitute (under specified conditions) a system of external relations between the same objects. It is this dualism which provides the promised bridge between the two groups of present mathematical structuralists.