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The Flight from Intuition Revisited

Abstract

We, historians and philosophers of mathematics, correctly view the 19th Century developments in geometry, algebra and set theory as flights away from intuition. These trends rejected the traditional idea that mathematics requires a sort of intuitive grasp of its content – a special mathematical perception. Kant's constructivist philosophy of mathematics forcefully and influentially expressed that traditional idea; so we rightly take the flight from intuition as a turn against Kant. Further, we correctly see post-19th-Century formalism and the modern emphasis on structure as outgrowths of those anti-Kantian trends.

My talk aims to show, however, that when we look closely, we will find a more nuanced situation with respect prominent formalist and structural trends. I will demonstrate that these trends indeed reject Kant's strict connection between all of mathematics and constructive intuition, and yet (each in its own fashion) remains sensitive to a Kant-like notion of intuition. Specifically:

A. Hilbert's 20th Century formalism grew from developments in geometry, and was committed to the full force of set theoretic reduction. Nevertheless, I will show that Hilbert's Program rests on a robust, indeed Kantian, notion of intuition for its 'finitary' foundation of mathematics.
B. Contemporary Category Theory, which arose from algebraic developments, primarily studies structure and structures. However, I will argue that Category Theory itself is tied up with a type of intuition. This will be a non-constructive notion of intuition, one that comports well with our common-sense understanding of that term. It is a notion that goes beyond the strictly Kantian notion; but it will be a notion of intuition that nevertheless performs a role for Category Theory quite analogous to the role that Kant and Hilbert envisioned for the constructive notion of intuition.

Point (A) is mainly historical and comprises the first two parts of the talk. Point B is speculative and systematic; I take it up in Part III. All of this will rest on a central distinction between what a theory (empirical or mathematical) is ***about***, versus what it is ***sensitive*** too.

On the historical side, Part I derives that central distinction from Kant and shows the way in which, for Kant, empirical science is sensitive to perception, and mathematics is sensitive to a more abstract notion of intuition. This will allow us precisely to define the modern geometric, algebraic and set theoretic flights from Kantian intuition. Part II sketches the delicate way in which Hilbert and his coworkers provided a foundation for set theory (and other aspects of post-Kantian mathematics) that is nonetheless sensitive to the Kantian notion of intuition.

On the systematic side, Part III will suggest a broader notion of intuition that preserves the leading aspects of Kantian intuition, but in a non-constructive manner. It will show the precise way in which Category Theory is sensitive to this broader notion of intuition.

At the end, I will suggest that these historical and systematic considerations address a contemporary debate about the foundational roles of set theory versus category theory.