Neo-logicism, structuralism, and Frege's applications constraint

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ABSTRACT

Most contemporary work in the philosophy of mathematics concerns what is practiced by professional mathematicians, or at least the mathematics taught in graduate and advanced undergraduate courses. Philosophers address ontological, epistemological, and methodological issues concerning branches like number theory, real analysis, complex analysis, functional analysis, and the like. Much, but not all of the work focuses on foundational branches, such as set theory

Contemporary philosophers are also keenly interested in the *applications* of mathematics. Typically, that concerns the use of mathematics in science, such as its role stating physical laws, the place differential equations in physics, and the like. There is less concern, overall, with the role and place of very elementary mathematics in everyday life, such as the use of numbers to count various collections, to balance checkbooks, to and measure medicine and wood.

Of course, it is assumed that there is *some* connection between the everyday use of numbers to count and measure—say a statement that I have three children and that the front of a given boat is 8 meters across—and the use of numbers in advanced mathematics, but the exact nature of this relationship is not often queried.

One exception to the trend of focusing on higher-mathematics is Scottish neo-logicism. The program began with Crispin Wright's seminal *Frege's conception of numbers as objects* [1983] and was extended with Bob Hale's *Abstract objects* [1987]. It continues through many extensions, objections, and replies to objections.

The overall plan is to develop branches of established mathematics using abstraction principles in the form:

$$\forall a \forall b(\Sigma(a) = \Sigma(b) \equiv E(a, b))$$

where a and b are variables of a given type (typically first-order, ranging over individual objects, or second-order, ranging over concepts or properties), Σ is a higher-order operator, denoting a function from items of the given type to objects in the range of the first-order variables, and E is an equivalence relation over items of the given type.

The neo-logicists argue that only their account respects Frege's so-called *applications constraint*. The idea is that a philosophical account of a given mathematical theory has to build in the typical applications of the theory. As Crispin Wright puts it, echoing Frege himself, "a satisfactory foundation for a mathematical theory must somehow build its applications, actual and potential, into its core—into the content it ascribes to the statements of the theory—rather than merely 'patch them on from the outside"'. The argue that the structuralist fails to respect this, claiming that we "change the subject", delivering isomorphic imposters to theories like elementary arithmetic.

The purpose of this talk is to assess the relevance of the applications constraint, arguing that it goes directly against a marked trend in mathematics, away from intuition and away from applications. Moreover, a close analysis of how the constraint is supposedly satisfied in neo-Fregean logicism shows that, thanks to some standard work due to Richard Dedekind, both structuralism and neo-logicism meet (or fail to meet) the constraint in a remarkably parallel manner.