Non-Eliminative Structuralism, Fregean Abstraction, and Non-Rigid Structures

Mathematical structuralists who think that abstract structures exist face a problem. The problem comes from the existence of non-rigid structures, structures that admit non-trivial automorphisms. A non-trivial automorphism is an isomorphism from a structure to itself that is not the identity mapping; under the automorphism, there is an object in the structure that is not mapped to itself. Non-trivial automorphisms create an identity problem for mathematical structuralism. According to some, structuralism entails that objects in a structure with exactly the same properties are identical. Non-trivial automorphisms on a structure show that the structure has two distinct objects with the same properties. The structuralist is thus forced to identify objects that should be distinct. This is the problem of non-rigid structures.

The problem of non-rigid structures is a general problem threatening several versions of non-eliminative structuralism. This paper offers a solution to the problem of non-rigid structures for a particular version of non-eliminative structuralism based on Fregean abstraction principles, as developed by Øystein Linnebo and Richard Pettigrew (2014). In this context, abstraction is used as a tool that can be applied to a system of objects in order to obtain the pure abstract structure of that system.

Our solution to the non-rigidity problem argues that a precise specification of a system should at least indicate which objects in the domain of the system are and are not identical with one another. The most straightforward way to indicate the identity and non-identity of objects in the domain of a system is to explicitly include relations involving identity in the specification of the system. We examine various ways to include relations involving identity in the specification of a system with the goal of solving the non-rigidity problem.

We first show that simply adding the two-place identity and non-identity relations to the specification of a system does not solve the problem. In fact, the same non-trivial automorphisms that cause problems for systems not including these relations continue to cause problems when the relations are included in the systems' specifications. A solution to the non-rigidity problem therefore requires more than simply adding the two-place identity and non-identity relations. The solution we propose is to include a number of one-place relations (i.e., monadic properties) to the specification, one such property for each distinct object in the domain. Specifically, for every system S there is a system S^* that expands S so that for every object in the domain, there is a monadic identity relation included in the specification of the system S^* that is instantiated by that object only.

We show that adding individual monadic identity relations for each object in the domain solves the non-rigidity problem. We conclude by arguing that this solution is philosophically acceptable for this version of non-eliminative structuralism.

References:

Linnebo, Ø. and Pettigrew, R. (2014). Two types of abstraction for structuralism. *Philosophical Quarterly*, 64:267 – 283.