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Abstract I shall begin by making some comments on Hilbert's methodological point of view in his first essay on proof theory of the 1920s 'A New Grounding of Mathematics' (1922). In number theory (to begin with), he proceeds from signs qua concrete, intuitable and irreducible objects. With these he operates, but makes also contentual statements about them. Hilbert is, of course, aware that the whole of number theory and real analysis cannot be constructed by means of contentual, intuitive methods. These methods break down when we are to deal with statements about functions or infinitely many numbers. It follows that from Hilbert's finitist point of view in the early 1920s, universally quantified sentences cannot appropriately be analyzed as possibly infinite conjunctions, but must be introduced axiomatically. In order to be able to prove effectively and successfully the consistency of the axioms of second-order arithmetic, Hilbert suggests the following strategy: All statements of classical analysis are converted into formulae that can be concretely exhibited. In other words, the entire mathematical theory, including its axioms, is formalized. Formalized mathematics is joined by contentual, informal metamathematics which serves the sole purpose of carrying out a finitist consistency proof for a formalized mathematical theory *T*. One problem that emerges from Hilbert's formalist approach in the 1920s is this. On the one hand, he regards both the mathematical and the logical signs and operations as detached from all meaning once the process of formalization has been completely carried out. On the other hand, there are several places where he characterizes the finitary or real sentences, in contrast to the transfinite or ideal sentences, of formalized arithmetic expressly as meaningful (Hilbert 1926, 1928). But why should it matter that in formalized arithmetic meaningful sentences be derivable at all? I make one proposal why for Hilbert it might still be of interest to be able to rely on meaningful sentences in the language of formalized arithmetic. Another problem that arises from his formalist approach in the 1920 is the fact that his transfinite axioms fail to provide any formal explication of the term "infinite". In the second part of my talk, I shall make some comments on the formalization of metamathematics in Hilbert & Bernays, *Foundations of Mathematics* (vol. 2, 1939) in the light of Gödel's incompleteness theorems and Gentzen's 1936 purportedly finitist (or at least constructive) consistency proof for Peano Arithmetic. In doing so, I place special emphasis on what Hilbert and Bernays still seem to regard as finitistically admissible proof-theoretic means, such as transfinite induction as applied by Gentzen in his proof.