Formalism and (set theoretic) truth

An initially attractive formalist position on set theory is that it is

simply a symbolic game, an axiomatic formal system developed to encode our mathematics of limits and infinite structures (which itself developed over history as an informal symbolic game). A major problem with this view is that, by Goedel's theorem, no axiomatic theory can derive all that we intuitively take to be true about even elementary mathematics. For example, intuitively, there is a fact of the matter about each truth of arithmetic, and all these truths should remain true within the set theory of the natural numbers; but this cannot be if set theory is a formal axiomatic system.

A natural response to this is to utilise infinitary logic. We could attempt to characterise set theory as an axiomatic theory with an \omega-rule, i.e. the usual set theoretic axioms together with an rule that says if Fn is derivable for each finite ordinal n, then \forall x\in\omega Fx is also derivable.

Of course making use of such an \omega-rule presumes the very thing a game formalist sets to explain: infinite sets. I will argue that formalists can avoid this circularity by using something weaker than the \omega-rule, the recursive \omega-rule, to obtain a rich axiomatization of set theory. The recursive \omega-rule says if there is a recursive function with outputs a derivation of Fn for each finite ordinal n, then \forall x\in\omega Fx is also derivable.
A formalist can then argue that truths of set theory are formulae derivable from the axioms of set theory and this additional rule.

I will then discuss in what sense Goedel’s theorem reapplies to this approach and its significance to second order logic and the meanings of the logical and set theoretic constants.