

Formalism in the Face of Complex Numbers

In his seminal 1960 paper, the physicist Eugene Wigner described the effectiveness of mathematics in physics as a *miracle*, which we neither *understand* nor *deserve*. Following members of the Hilbert School, Wigner's views of mathematics are characteristically formalist. He defines mathematics as "the science of skillful operations with concepts and rules that are invented *only* for this purpose".

In this paper, I aim to show that this formalist conception of mathematics, which many of Wigner's contemporaries shared and eventually became the dominant philosophy of mathematics in the 20th century, has many shortcomings. It is this formalism that creates miracles *out of the thin air* (Ferreirós) and turns the applicability of mathematics in the natural sciences into a *happy accident*. (Unger and Smolin, also Colyvan, Lützen, Grattan-Guinness) More importantly, this conception of mathematics, as I will show, gives an inaccurate picture of how a concept is first developed and how it evolves in time. As a result, it is unable to account for the transition that the mathematics community goes through (sometimes over the course of centuries) with respect to the acceptance of a new concept.

My methodology is based on a historically sensitive study of complex numbers, which according to Dirac, Feynman, Wigner, Steiner, and Penrose among others, present us with one of the most difficult cases of the applicability. Contrary to Wigner, I aim to show that complex numbers, studied through their historical development present a case against this kind of formalism. As I will show, the introduction of complex numbers in the 16th century was faced with skepticism and resistance, and the mathematicians who decided to work with them adopted a more nominalist attitude. The transition to a more realist point of view was facilitated by factors such as fruitfulness, novel results and generality (having real numbers as their limiting case). As we will see, moreover, besides developing the proper algebra for complex numbers, finding their geometrical representation as vectors, or points on a two-dimensional plane played a crucial role in this transition. What made complex numbers useful to the physicists of the 20th century wasn't their manipulability in Cardano's formula (in the 16th century) but their connection to trigonometric functions through Euler's identity and ultimately their use in the Fourier series.

Using Carnap's terminology, the case of complex numbers shows that the transition with respect to the acceptance of these new concepts was achieved not through *external*, philosophical debate about their existence or their legitimacy as numbers but based on *internal* mathematical work, some of which had substantial result for their applicability of in the natural sciences.