Intuition and the end of all -isms

For historical reasons, one might see formalism and structuralism as defined by their opposition to any kind of *intuitionism* in mathematics whereby, at first, by intuition, Kantian *constructions* in space and time were meant. But such a negative delimitation comes at a price, typically leading to formalism's sudden metamorphosis into theories which have intuition built into their own foundations. So, e.g., Frege's and Dedekind's logicism treated arithmetic as substantially dependent on the axiomatic – i.e. formula-producing – systems in which no recourse to intuition was allowed. In the end, though, they led to type theory with its constructively built up universe. Similarly, Hilbert's early axiomatism – as a reaction to the advance of non-Euclidian, i.e. counter-intuitive geometries – gave birth to the formalist interpretation of axioms and their reading as implicit definitions of mathematical structures. But it ended up with the concept of the 'finite Einstellung', in which the manipulation with symbols is to be controlled by a direct intuition. In light of this, even the 'revolution' of Brouwer does not seem to be such a radical break with classical logic and set theory but instead represents an explicit acknowledgment of their tacit preconditions to which the systematic use of constructive principles such as transfinite induction or situation-dependent formations such as diagonalization belong.

To say that the confusions described above grow out of an ambiguous concept of intuition is both true and trivial. Of real interest, in fact, is the source of the whole shift in which – to paraphrase Einstein – all of the related -isms became the same. In my paper, I would like to identify this source with the gradual tendency to stress the practical rather than the subjective dimension of intuition in accord with Kant's reference to *constructions* in intuition as being the very source of mathematical truth. By this 'pragmatic turn', so to speak, some traditional dilemmas such as that between mathematical realism and nominalism (is the meaning of arithmetical signs some object existing beyond the sign, be it the abstract number or mental construction of it, *or* the sign "5" itself?) were solved. But others were arising simply because, by 'practical', a lot of things can be meant as the case of the word "effective" or "effectively calculable" has shown in the context of theorems such as the Church-Turing thesis. What I claim is that this pragmatic turn brought the philosophy of mathematics a bit closer to the modern philosophical debates and freed it, in fact, from its earlier Kantian bounds.

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