Structuralism as a philosophy of mathematics – what it is about?

It is difficult if not impossible to account for all mathematical disciplines in the boundaries of one philosophical theory. Usually theories that offer a plausible account of abstract theories are less plausible when we turn to applied mathematics or mathematical physics. It seems reasonable to discriminate three levels of complexity of mathematical theories. Structuralism seems to be a plausible account of one of these three levels, but offers only a partial account of the remaining two of them. Thus the proper question to ask about structuralism is not whether it is plausible or not, but which aspects of mathematics does structuralism explain and which it does not. In the paper I will discriminate three levels in mathematics.

The first level concerns ***idealizations***. There are two – the ***Euclidean idealization of forms***, such as numbers, geometric figures, algebraic structures, or sets (they are subject to mathematical idealization – they are a-temporal, a-causal, immaterial) ant the ***Newtonian idealization of dynamical systems*** – mechanical, thermodynamical, electrodynamical, quantum. All these are subject of physical idealization, they use the concept of state (i.e. are a-historical), the changes of which are described by means of differential equations (i.e. are non-adaptive). If we compare physics with mathematics, we see that mathematics uses a-temporal, a-causal, immaterial idealities, while physics uses temporal, causal and material idealities. One way to understand structuralism is as a theory of idealization.

The second level of changes concerns the ***instrumental level***. Mathematical objects are not accessible directly, but mathematics creates instruments of symbolic and iconic representation, by means of which we can study these objects. On the symbolic side we have instruments like the decadic positional system of arithmetic, the symbolism of matrices used in linear algebra, etc., while on the iconic side we have the constructions by ruler and compass in synthetic geometry, the coordinate system in analytic geometry, etc. Despite the fact, that numbers as objects of mathematics are a-temporal, a-causal, and immaterial, the numerals are pieces of ink on paper, thus they are temporal (we can manipulate them), causal (we can perceive them), and material. Structuralism can be interpreted as the theory of objects constituted by such an instrumental practice.

The third is the level of ***conceptual changes*** that occur inside of the realm accessible by a particular instrument. Thus if we restrict ourselves for example to ruler and compass constructions as the basic representational means, we can witness still a considerable evolutionary dynamics along the line connecting Euclid’s Elements, projective geometry, non-Euclidean geometry, Beltrami’s model, Klein’s classification of geometries, Poincaré’s combinatorial topology. It seems that structuralism first emerged at this third level, when mathematicians like Riemann, Klein, Dedekind, and Noether started to see their subject as the study of structures. This approach as a way of doing mathematics received its first conscious expression in van der Waerden’s seminal *Abstract Algebra* and reached its culmination in the monumental achievement of Bourbaki.