Wittgenstein against Formalism

In his manifesto, "On the Infinite," David Hilbert proposes to defend infinitary mathematics, including Cantorian set theory, from charges that it is false and even perhaps inconsistent--in the latter case it could infect all of mathematics, which is embedded in it. His strategy involves regarding all infinitary mathematics as a formal system (so that the issue of truth and falsity do not arise), backed up by a formal consistency proof. He declares: "No one is going to turn us out of the paradise which Cantor has created."

In his 1939 Lectures on the Foundations of Mathematics at Cambridge, Wittgenstein scoffed at the need for such a program: "I wouldn't dream," he said, "of trying to drive anyone out of this paradise. I would try to do something quite different: I would try to show you that it is not a paradise-so that you'll leave of your own accord. I would say, 'You're welcome to this; just look about you.'"

Above all Wittgenstein ridiculed the need to prove that mathematical systems are consistent before they can be used. Here, however, he met stiff opposition by Alan Turing, who attended Wittgenstein's lectures. Turing declared that Wittgenstein was forgetting that mathematical systems have physical applications. An inconsistent system could cause "bridges to fall". Wittgenstein was going far too far. Wittgenstein hotly denied this.

In this lecture we will see why Wittgenstein took these positions, and will attempt to see whether Wittgenstein or Turing had the upper hand in the argument. We will also see why Wittgenstein held Goedel's theorems to be irrelevant to the issues discussed (though almost everybody else thinks they are).