Formalization and Justification

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Philosophers often distinguish between two types of proofs. *Informal proofs* are what appear in textbooks and journal articles, whereas *formal proofs* are finite sequences of formulae (in the language of ZFC, PA, or some canonical system) such that each formula is either an axiom or follows from previous formulae by mechanical rules of inference.

According to a common view, informal proofs indicate the existence of a formal proof [Azzouni, 2004, 2009, 2013].[1] Further, Burgess [2015, p. 90] argues that so-called *rigorous* informal proofs indicate the existence of formal derivations by providing "enough steps." Thus, many philosophers have argued that the relationship between formal and informal proofs is crucial for characterizing (1) what an informal proof is and (2) what makes it rigorous.

But what is the relationship between formalization and rigor on one hand, and justification on the other? Clearly, having a proof of a theorem T is not necessary for justifiably believing T. If Maryam Mirzakhani endorses some proposition of ergodic theory, then math enthusiasts might justifiably believe that proposition without proof. Alternatively, probabilistic "proofs" and empirical methods might provide sufficient justification for believing a theorem in the absence of proof [Baker, 2009, Fallis, 1997]. But if understanding a proof is merely sufficient for justification, does the justification provided by proofs differ in strength or kind from that provided by expert testimony or experimental methods?[2]

Many philosophers have argued that proofs provide a *greater degree* of justification than do non-deductive methods.[3] Modern philosophers often held this

[1] Burgess [2015, p. 31] also attributes this view to Hyman Bass. Avigad [2010] similarly claims, "It is sometimes helpful to think of ordinary mathematical proofs as being higher-level descriptions of low-level formal axiomatic proofs, or recipes for constructing such proofs."

[2] Easwaran [2009] argues that proofs are *transferable* in a way that non-deductive methods are not. However, Easwaran does not argue that transferability increases an individual's justification for believing a theorem; he writes, "However, transferability is a social norm – it can help the community develop a better grasp on the knowledge of its members, even though it may not have any advantages for the individual."

[3]Some even claim that mathematical proofs produce certainty. For instance, [Frege, 1980, p. 4]"The aim of proof is, in fact, not merely to place the truth of a proposition beyond all

view because, very roughly, they thought (i) the justification conferred by an argument is derived from how "clear" or "evident" the argument makes its conclusion, and (ii) mathematical proofs are step-by-step arguments in which successive steps are "self-evident."[4] Philosophers and logicians now might identify a "self-evident" step with the application of a valid, recursive rule of inference [Sieg, 2009]. One might infer that formal proofs provide the greatest possible degree of justification because all steps in a formal proof are explicitly justified by sound and recursive rules of inference. Hence, if informal proofs indicate the existence of formal ones, then informal proofs might likewise confer a maximal degree of justification. I reject this argument.

This paper argues that informal proofs often provide greater justification for believing a theorem than do formal derivations. After doing so, I consider and reject arguments from the early twentieth century that formal proofs are more secure than informal ones.

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doubt, but also to afford us insight into the dependence of truths upon one another." A similar emphasis on certainty is present in modern philosophy, in particular, Descartes' writings.

[4]For instance, Locke writes,"[I]it is plain that every step in reasoning that produces knowledge, has intuitive certainty ... [T]o make anything a demonstration, it is necessary to perceive the immediate agreement of the intervening ideas, whereby the agreement or disagreement of the two ideas under examination ... is found" [Locke, 1975, Book IV, Section 2, vii]. In the twentieth century, Gödel writes, "the outstanding feature of the rules of inference being that they are purely formal, i.e., refer only to the outward structure of the formulas, not to their meaning, so that they could be applied by someone who knew nothing about mathematics, or by a machine" [Godel, 1933].

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